Elastic Instability (Buckling)

Worked Example 1 – Pin-Jointed Truss Structure

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MMME2053 – Mechanics of Solids

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Worked Example 1

Truss Pin-Jointed Structure (From 1617 Exam Paper)

A rigid beam is supported by a truss pin-jointed structure, as shown in Figure Q5. It is subjected to a point force of magnitude F with direction orientated at 45° to the horizontal plane as shown.

i) Indicate which members are in tension and which are in compression.

ii) Indicate the most critical member due to buckling (justify your answer by calculation).

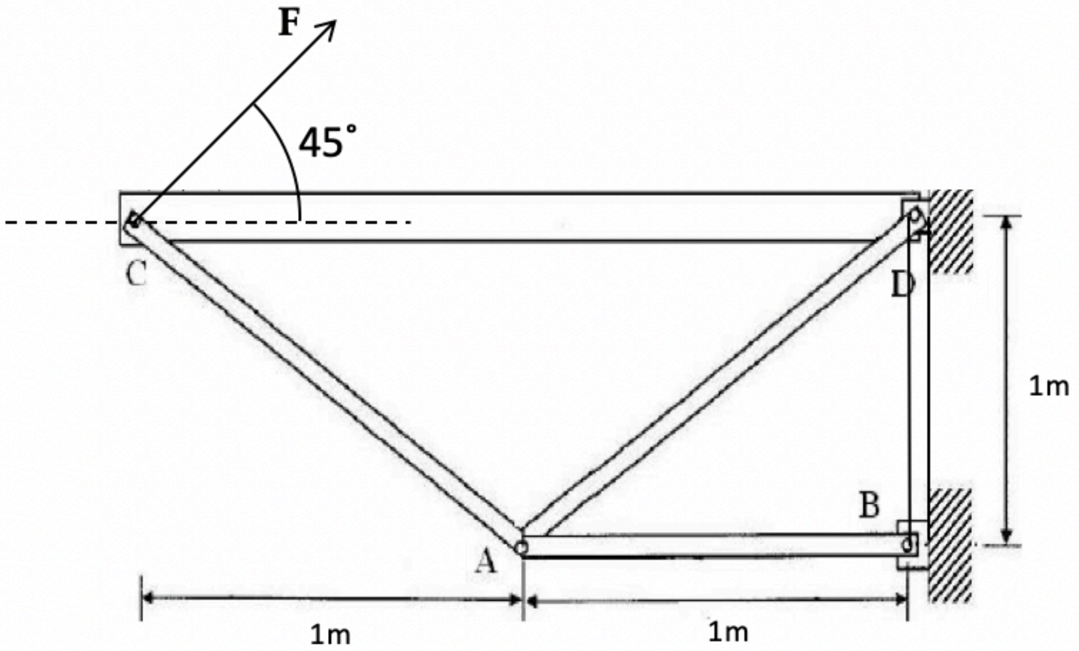
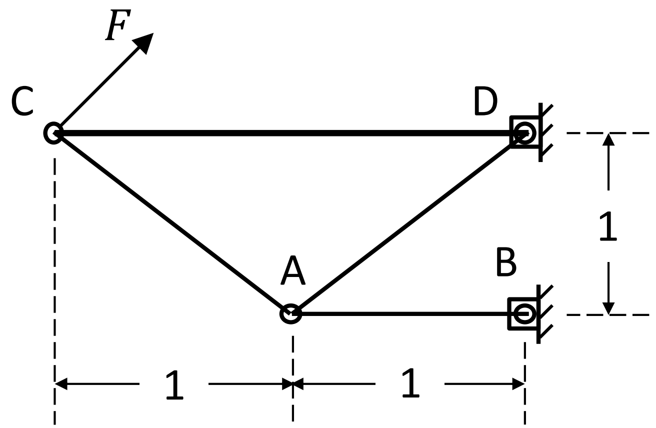


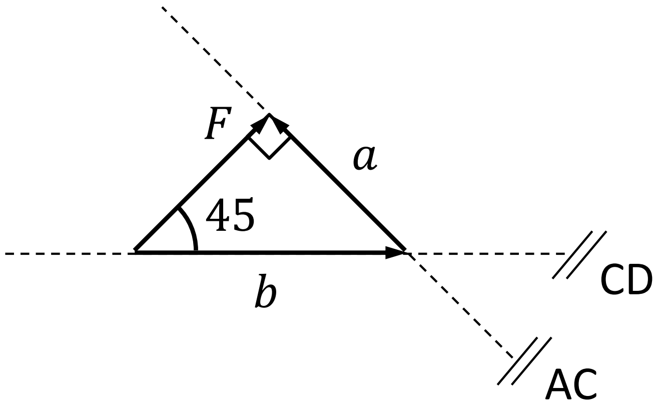
Figure Q5

Labelling the Structure & Force Calculation



Section BD has been removed from the diagram as there is no force in this member.

Forces are in equilibrium; therefore, force polygons can be used. Drawing a force polygon for the force applied at C:



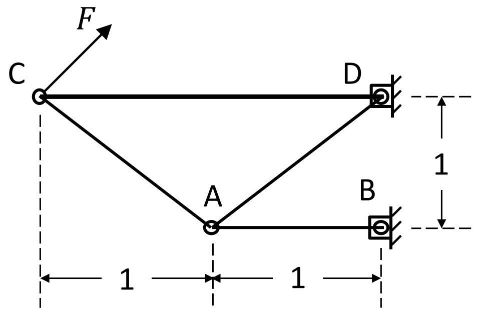
From geometry, it can be seen that this is an equilateral triangle. Therefore,

$$a = F$$

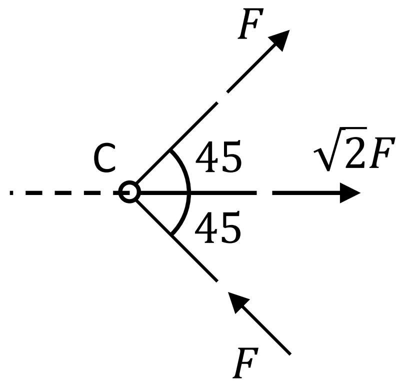
and resolving onto CD:

$$\cos 45 = \frac{F}{b} \qquad \therefore b = \frac{F}{\cos 45} = \sqrt{2}F$$

Critical Loads



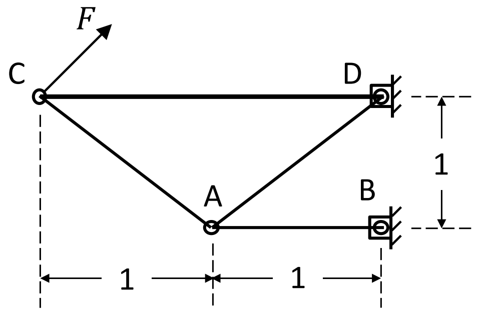
Using the results from the force polygon to draw a free body diagram at joint C:



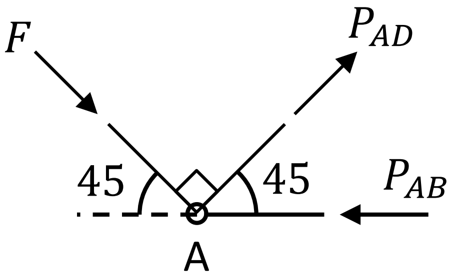
Critical load in member AC: $$P_c^{\,AC} = \frac{\pi^2 EI}{L_{AC}^{\;2}} = F \qquad \therefore F = \frac{\pi^2 EI}{L_{AC}^{\;2}} = \frac{\pi^2 EI}{\left(\sqrt{2}\right)^2} = \boldsymbol{\frac{\pi^2 EI}{2}}$$

Critical load in member CD: $$P_c^{\,CD} = \frac{\pi^2 EI}{L_{CD}^{\;2}} = \sqrt{2}F \qquad \therefore F = \frac{\pi^2 EI}{\sqrt{2}L_{CD}^{\;2}} = \frac{\pi^2 EI}{\sqrt{2}\times 2^2} = \boldsymbol{\frac{\pi^2 EI}{4\sqrt{2}}}$$

Critical Loads



Free body diagram at joint A:



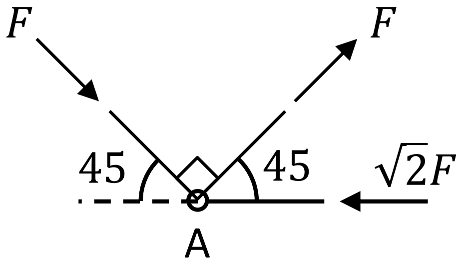
Vertical equilibrium gives: $F sin(45) = P_{AD} sin(45)$

$\therefore P_{AD} = F$

Horizontal equilibrium gives: $P_{AB} = F cos(45) + P_{AD} cos(45)$

$\therefore P_{AB} = F cos(45) + F cos(45)$ $\quad\quad$ $\therefore P_{AB} = \sqrt{2} F$

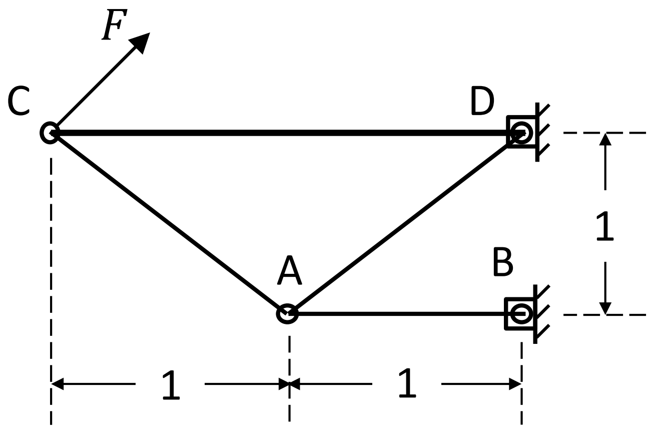
The FBD can therefore be redrawn as:



Critical load in member AB: $P_c^{AB} = \frac{\pi^2 EI}{L_{AB}^2} = \sqrt{2} F$ $\quad\quad$ $\therefore F = \frac{\pi^2 EI}{\sqrt{2} L_{AB}^2} = \frac{\pi^2 EI}{\sqrt{2} \times 1^2} = \boldsymbol{\frac{\pi^2 EI}{\sqrt{2}}}$

Critical load in member AD: $P_c^{AD} = \frac{\pi^2 EI}{L_{AD}^2} = F$ $\quad\quad$ $\therefore F = \frac{\pi^2 EI}{L_{AD}^2} = \frac{\pi^2 EI}{(\sqrt{2})^2} = \boldsymbol{\frac{\pi^2 EI}{2}}$

Critical Member



Critical load in member AB: $F = \dfrac{\pi^2 EI}{\sqrt{2}}$

Critical load in member AC: $F = \dfrac{\pi^2 EI}{2}$

Critical load in member AD: $F = \dfrac{\pi^2 EI}{2}$

Critical load in member CD: $F = \dfrac{\pi^2 EI}{4\sqrt{2}}$

Therefore, since members AB and AC are in tension, these will not fail due to buckling. Of the other two members, which are both under compression and therefore potentially subject to buckling, **member CD requires the smallest force to cause buckling and so is the most critical member due to buckling**.